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IS APPROVAL VOTING AN "UNMITIGATED EVIL?" A RESPONSE TO BRAMS, FISHBURN, AND MERRILL

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IS APPROVAL VOTING AN "UNMITIGATED EVIL"?: A RESPONSE TO BRAMS, FISHBURN, AND MERRILL

Abstract. Brams, Fishburn, and Merrill [3] argue that the indeterminacy of approval voting (AV), introduced in our earlier paper [11], is not a vice, but a surpassing virtue of AV. BFM did not analyze the negative vs. the positive features of AV, so their assertion remains a conjecture. In our response, we discuss the issue of evaluating and balancing the costs of AV against any merits. Moreover, by answering BFM's comments, the argument against AV becomes stronger. This is because the region of AV's indeterminacy is quite large. We show that this means the AV outcome can negate the voters' true wishes, that AV can be volatile even to minor fluctuations of voter behavior, and that AV is one of the most susceptible systems to manipulation by small groups of voters (e.g., the AV outcome could be determined by small, maverick groups). Under specific circumstances, AV may be appropriate. To identify these situations, we propose the more accurate name of the "Unsophisticated Voter System".

We welcome this opportunity to add commentary about approval voting (AV) to our more general and essentially technical paper S&VN [11]. When we followed the suggestion of S. Brams and others to analyze AV with a new methodology (developed in [7,8,9] and based on concepts coming from "dynamical chaos"), we thought our conclusions might indicate that, in some manner, AV is better than most other systems. We knew we would discover new faults of AV - all systems have them. But, we wondered whether these flaws could be countered by other, more favorable traits. In retrospect, we probably were counting on the appealing dream expressed by Brams, Fishburn and Merrill (BFM) [3] that "*..it seems rational for [a voter's] approval ballot to indicate considerations of not just ordinal but also cardinal information.*" However, as the history of social choice has proved so often, you need not get what you want. Their wish seems rational; the side effects of AV are not. As we showed in S&VN, AV's ability to account for the voter's intensity of preferences induces "indeterminacy"; this means the AV outcome can negate the voters' true wishes.

In S&VN we analyzed all possible multiple voting systems, so AV was treated, essentially, as one of several examples. Consequently, in S&VN we did not compare AV with other multiple voting system, and we did not describe other AV properties related to its indeterminacy. To adequately respond to BFM, we now present some of

these results. (Still more AV properties could be given, but, here, we discuss only results answering BFM's comments.) The thrust of these more specific comments prove that AV is worse than implied in S&VN. This is because the likelihood of indeterminacy is greater than suggested in S&VN (so there is a better chance the AV outcome can negate the voters' wishes). As a direct consequence of its indeterminacy, the AV system is more sensitive to even minor fluctuations in voter behavior and more susceptible to being manipulated by small groups of voters than any standard procedure. These traits did not surface in S&VN because the more negative features of AV were obscured by our general discussion of all multiple systems. Even among multiple voting systems, AV gets low grades.

Why should we consider a voting system with defects so serious that they can violate the purpose of an election? We don't know, but there may be reasons. If so, it is incumbent upon an advocate of AV to spell out not only AV's pluses but also its minuses, to carefully analyze the tradeoffs, and to justify why we should believe it is "rational" to use a procedure that can defeat the purpose of an election. If BFM's support for their statement "*Our response .. is that (S&VN's) indeterminacy is not a vice but instead a surpassing virtue of AV*" had moved the debate in this direction, we would not have responded. However, they "brushed off" the serious side effects of AV by asserting that the negative aspects of indeterminacy will not occur with AV. Their justification primarily is based on concocted "scenarios" or specific examples rather than analysis. Therefore, they did not analytically compare AV's "vices" versus its "virtues" so we could see whether "*.. the proposed cure (of AV) is not worse than the disease.*" (S&VN) BFM do raise interesting issues about strategic behavior. In our response, we move the discussion to the main point. Namely, before using AV, we need to know what we are getting; we need an *analytic* study of AV over *all* profiles to determine whether AV's associated costs can be balanced by any positive features.

Before starting our response, we clarify some of their comments. The reader familiar with S&VN may be confused by BFM's assertion that "*.. the Borda system [is] a determinate voting system favored by Saari and Van Newenhizen*" when we specifically stated that the Borda Count (BC) is treated only as a technical condition or as an example. (The favorable properties of BC do place BC in any list of top contenders for the "best system", but we do not know whether it is "the best".) Similarly, their statements about strategic voting may have raised questions. The explanation may be simple. When BFM refer to BC, they mean results

in Saari [8] and a revised version. For strategic voting, we suspect they refer to Saari [10]. We discuss results from [10] relevant for our exchange of opinions.

Secondly, BFM praise our reluctance to anoint any method as being the "best" reform alternative to replace the plurality vote. They then claim this is inconsistent with our questioning of AV. It is not. Compare the search for the "best" reform procedure with a search for the "best choice" of anything - say, the purchase of a new car. Although subtle features can complicate the final choice of the "best" vehicle, often it is easy to dismiss other vehicles from consideration because of serious faults. Of course, a dismissed, flawed vehicle may have hidden features to compensate for its defects, but the burden of proof is on the car salesman to prove this. In the same spirit, now that we know AV has serious defects, maybe we should look elsewhere for an "election reform vehicle". Although we could reconsider AV, it is rational to do so only after an "AV salesman" produces a careful, convincing bottom line analysis. But, while we believe such an accounting is necessary, we did not, as BFM assert, ever voice the "..*opinion that AV's indeterminacy is an unmitigated evil.*" This may be true, but in S&VN we could not endorse such an unqualified statement. We leave it to the reader to determine, after reading S&VN, BFM, and our response, whether their phrase more accurately describes AV than ours.

2. THE OMNIPRESENT DANGER OF INDETERMINACY

An AV election can be indeterminate. The idea is this. Suppose we know each voters' rankings of the candidates. Armed with this information, we can determine the unique election outcome. Not with AV. With AV, even with the same sincere voters, there could be many different outcomes. Because of this *indeterminacy*, we are forced to wonder which AV outcome reflects the group's true wishes. Indeed, how do we know that the actual AV outcome doesn't completely frustrate the real choice of the voters? A more disturbing situation is **complete indeterminacy** where *all* possible outcomes can occur with the same sincere voters. In other words, if the candidates are A, B, and C, then there are situations where, with the *same sincere voters*, the election outcome could be A>B>C, or B>C>A, or C>A>B, or B>A>C, or C=A>B, or ... The voters never alter how they rank the candidates; the wild variations in the election outcomes occur as the voters vary in how many candidates they approve. In other words, these various outcomes occur

with sincere, knowledgeable voters; they are not based on capricious, irresponsible actions. In S&VN we demonstrated this phenomenon with an example that has been used elsewhere [8]. This is where 6 voters have the ranking A>B>C, 5 voters have the ranking C>B>A, and 4 voters have the ranking B>C>A. It is easy to show that all thirteen possible rankings can occur depending on whether the voters mark approval for their first, or their first and second ranked candidates. Thus, while other methods have flaws, you need to change profiles to find them; *AV gives you all possible flaws with the same profile.*

Election rankings

Are there tradeoffs to justify the defect of indeterminacy? To answer this question, we need to know whether indeterminacy is a serious problem. After all, one might conjecture, as BFM claim, that the indeterminacy of AV occurs only in bizarre situations. One way to judge whether a situation is "bizarre" is to follow the lead of BFM by using standard election rankings. With this approach, any statement of BFM's kind is totally false. We showed in S&VN that *AV's complete indeterminacy can accompany* ***all possible*** *election rankings, for* ***all*** *subsets of candidates, obtained with* ***any*** *standard procedure.* In other words, no matter what is the procedure, what are the subsets of these candidates, and what are the election rankings, the associated AV election ranking can be completely indeterminate. In direct contrast to what BFM assert, the facts are that the *indeterminacy of AV can accompany* ***all*** *possible situations;* it is omnipresent.

The flexibility derived from the universality of AV's indeterminacy have interesting consequences. For instance, they allow *you* to use standard election rankings to define what you believe are reasonable measures of voter conformity. It follows from S&VN that even should your standards - Condorcet winner, BC election outcome, whatever they are - indicate that the voters agree on a particular candidate or ranking, the accompanying AV results still can be completely indeterminate. Thus, the AV election outcome can frustrate and even reverse what you accept as being the voters' correct choice. We can illustrate this with an example more extreme than most people would require. Suppose you accept that a group's true ranking is A>B>C *only* in the very special setting where not only is the group's election ranking (with the PV, the BC, or whatever system you want) A>B>C, but the same voters, by majority votes, also rank the pairs as A>B, B>C, A>C. Even here, the AV election outcome can be completely indeterminate, so an admissible AV outcome is the reversed C>B>A - or anything else. For four

candidates you might agree that the group's ranking is A>B>C>D only in the incredibly strict situation where A>B>C>D is the election ranking of the four candidates and where the election rankings for all four triplets and the majority rankings for the six pairs all are consistent with this ranking. Even for profiles with such strong harmony among the voters, the indeterminate AV outcome could be anything. Does AV's indeterminacy occur only in bizarre situations? No, it is omnipresent. Is AV's indeterminacy worrisome? Absolutely.

Our results have an obvious conclusion. Even if *all reasonable people agree the voters are in remarkable agreement about how to rank the candidates, the AV outcome can violate the will of these voters by being completely indeterminate!* Thus, the dream of incorporating a voter's intensity of preferences, can, at least under AV, turn into a nightmare. If we are asked to adopt, as an "instrument of democracy", a method that can radically distort even the strong, clear consensus of the voters, then we need exceptional assurances that what we get in return is worth the heavy price. These tradeoffs, emphasized in S&VN and central to whether indeterminacy is a vice or a virtue, are not discussed in BFM.

Profiles

Our results about AV's complete indeterminacy are not based on rare instances or highly concocted examples; BFM agree that "*..the likelihood of indeterminate profiles for AV is not at issue here.*" They accept the consequences of Theorem 6 in S&VN - our conclusions are robust and probable. Later in BFM they now assert that we didn't inquire "*..into the probability that these results ever would occur...*" We agree that this is a critical issue bearing on the suitability of AV, so we expand on our earlier comments in S&VN by discussing only AV. Intuition suggests - because AV can be indeterminate with any procedure, for any rankings for any subset of candidates - that *AV is indeterminate for most profiles.* It is. Indeed, the indeterminate behavior of AV is so close to being omnipresent over profiles that AV's indeterminacy is worse than many other multiple voting systems. These assertions can be justified in several ways. We decided that most experts could derive a strong, supporting, probabilistic statement by using the ideas developed in S&VN. (They involve simple algebraic conditions.) So, we felt it would be more useful to introduce a different, more intuitive approach. As a preparation, remember that in the tabulation of a ballot, weights, w_1, w_2,..., w_n, are selected to be assigned to a voter's top ranked, second ranked, ... , n^{th} ranked candidate. For instance, for the PV, $w_1=1$, $w_j=0$ for $j\geq 2$. The only

restrictions on the choice of these weights are that $w_i \geq w_{i+1}$ for i=1,..,n-1, and $w_1 > w_n$.

Theorem 1. A necessary condition for a profile *not* to admit indeterminate AV outcomes is that the *exact same* election ranking occurs for all possible choices of $w_1, w_2, .., w_n$.

In other words, the *only* time an AV outcome is *not* indeterminate is when all possible positional voting procedures have no flaws, they all give the exact same ranking. Thus, only in the extremely rare situations where there is absolutely no question with any positional voting system, no matter how bizarre, about the voters' rankings, then AV *might* escape indeterminacy; in answer to BFM, the region of AV's indeterminacy is very large. Theorem 1 is an immediate consequence of the next, easily proved statement that relates AV outcomes to the election rankings of simple methods. It asserts that even when the plurality outcomes - or the outcomes of any standard method - are judged to be perfectly reasonable, the AV outcome still could be indeterminate. To state this conclusion, note that to obtain an election ranking from the election tally, all we use is the ordinal information. This tally can be normalized so that the sum of the tallies for all candidates equals unity. Call this the *normalized tally*; the normalized tally expresses each candidate's fraction of the total tally. This values can be expressed as a vector where the j^{th} component indicates how c_j fared. For instance, the vector (1/6,1/2,1/3) corresponds to the ranking $c_2 > c_3 > c_1$ while (1/2,1/4,1/4) yields the ranking $c_1 > c_2 = c_3$. .

Theorem 2. Suppose there are n>1 candidates. Suppose the weights to tabulate the election, $w_1,..,w_n$, are given. Choose a profile of voters, and compute the normalized election tally. This tally is in the convex hull of the admissible, normalized AV tallies for the same voters.

Essentially, this theorem states that any election result, even if it is based on some bizarre, off-the-wall way to tabulate the votes, either is an admissible AV outcome or there is another, more extreme AV outcome. Clearly, the originators of AV did not intend AV to inherit all of the vices of all systems, but through its indeterminacy, as shown by Theorem 2, AV did. For instance, whenever a

profile has a plurality outcome that can be criticized, this outcome is one of several *indeterminate* AV outcomes. If a profile has a perfectly reasonable plurality outcome, but you can find some weird system doesn't agree, the AV outcome still is indeterminate! *Thus, the region of indeterminacy of AV includes all profiles with suspect plurality outcomes, along with many more profiles that have a perfectly reasonable plurality ranking.* If you, along with BFM and us, do not care for the PV, then replace PV with any other simple voting system and the same assertion about AV's region of indeterminacy holds. However, this extreme statement does *not* hold for all multiple voting systems - many multiple systems have a much smaller region of indeterminancy. Secondly, Theorems 1 & 2 are necessary conditions; they are not sufficient. For instance, in our original example, only A or C can be elected with standard procedures, but B along with A and C can be AV winners. So, the true extent of the indeterminacy of AV is greater than suggested by these theorems.

Theorem 1 answers BFM's question because it asserts that even if a bizarre method creates an election ranking different from the PV or the BC outcomes, the AV outcome already admits indeterminacy. Anyone experienced in constructing profiles for voting systems recognizes that Theorem 1 imposes a very strict conformity condition - in places, almost unanimity - on the voters in order to avoid AV's indeterminacy. Indeed, the voters could be only 1/10,000 away from unanimity and, yet, the AV outcome is indeterminate. To illustrate this, suppose out of the 10,000 voters ranking the candidates A, B, and C, 9,999 of them believe that A will do an excellent job, that B is quite mediocre but much preferred to C, and that C is an absolute disaster. Suppose the last voter prefers C, but believes B is much better than A. With these preferences, with AV, and using BFM's recommended strategy of mean utility, each voter votes for his or her two top choices. The AV tally for the first 9,999 voters is a tie vote between A and B. This tie is broken in favor of B when the last voter votes. So, excellence is the clear choice of these voters, but AV selects mediocrity. (Obviously, this indeterminacy is not "*..a surpassing virtue of AV.*") Moreover, such an outcome is almost unique for AV. Almost all other voting systems select A as the clear, robust winner; no small group of mavericks could alter the outcome.

A second consequence of this example and Theorem 1 is that it is difficult to avoid the indeterminacy of AV; it is nearly omnipresent. While we agree with BFM that "*..freedom may have a price in permitting a possibly dreadful outcome*", we

argue that the role of research in social choice is to minimize the cost, not to maximize it. The evidence indicates that AV does the latter.

Leadership and the MAA

The reason AV experiences difficulties when other systems don't is clear; AV only allows you to choose two groups - "the good guys" and "the bad guys". You cannot distinguish your first from your second from your third choice. Thus, it is easy to construct several robust examples where, because of indeterminacy, AV is almost unique in selecting mediocrity over excellence. If you are choosing leadership for your organization, this has to bother you. Consider L. Gillman's warning on the 1987 AV ballot for the Mathematical Association of America (This repeats his statement in [4].), "*..suppose there are three candidates of whom two are outstanding. Suppose the third is a person you believe is not yet ready for office but whom you decide to vote for as a means of encouragement (in addition to voting for your favorite). If enough voters reason that way, you will elect that person now.*" The vice of AV's indeterminacy creates a real danger an organization will not elect whom they really want. If the Math Association used a procedure that allowed the voters' to distinguish how they rank the candidates, Gillman's unusual warning would not have been necessary. Indeed, do you know of any other ballot for a professional society that included explicit warnings and proposed strategies so that the Association would not end up with a distorted outcome? It is a telling situation when *sincere* voters need strategies! This phenomenon is almost unique with AV, and it is reflected in BFM's emphasis on a voter choosing "*..his optimal strategy.*". This directly contradicts the impression one might get from BFM that AV reduces the need for strategic behavior - with AV you need strategies even to be sincere.

Scenarios

BFM claim that the negative features of indeterminacy won't occur with AV, and part of their argument is based on their scenarios. Most of us know that, with experience, we can start with most profiles and create the appropriate scenarios to justify almost anything. BFM aptly demonstrate this expertise by starting with our first example and constructing three different scenarios to justify three different AV outcomes, even though these outcomes reject BFM's usual measure that the Condorcet winner should be elected. As they admit, their scenarios "*..may be overdrawn and involve quasi-dichotomous preferences.*" (Namely, most of the voters use only "good guy" and "bad guy" categories.) On the other hand, it is equally

easy to create several other passionate voter scenarios with this same profile to demonstrate that the AV outcome is the wrong one. Again, with the same profile, one could create other scenarios, using only a few excitable voters, to model what the newspapers often call the small "lunatic fringe". Because only a couple of voters can alter the AV outcome, such a scenario suggests how vulnerable AV is to this fringe element. Of much greater importance, we should be more concerned about normal situations where the voters make reasoned distinctions about the candidates; how do you justify AV's indeterminacy here? So, with just *one* profile, we could construct many different scenarios; some support AV, most do not. Which scenario is the correct one? We admit we were tempted to base our response to BFM on our "stories". We resisted because scenarios don't prove anything. Scenarios, often based on highly concocted situations, are meant to illustrate potential issues. They cannot analyze "What else can happen?", "What are the tradeoffs?". A serious evaluation never is based on isolated examples, it must involve all possible profiles. This is done in S&VN; it is not in BFM.

For sake of argument, accept BFM's scenario approach. Why should we use AV to accommodate a couple of overdrawn scenarios, where even the Condorcet winner is rejected, at the expense that AV can violate the voters' wishes in the more prevalent, normal situations? This is a distorted tradeoff, so our answer to BFM is, yes, we do "..*wonder whether this kind of responsiveness is 'worth the accompanying cost of indeterminacy'.*" In other words, go beyond BFM's argument and accept the obviously false assumption that there are profiles where all associated scenarios indicate that the AV outcome is the best. This only proves there are situations where AV has a "plus". What about the many more situations, as indicated by Theorem 1 and other measures, where AV's indeterminacy can violate the voters' wishes? How do they all balance out? This analytic accounting is what we ask for. So far the analysis gives a negative answer for AV.

A positive feature of AV

What is the source of our differences of opinion about AV? Part is due to methodology. As BFM point out, the supporting analysis for their position is based on various assumptions, so it is based on certain, restricted subsets of profiles. Remember, when Brams and Fishburn started their analysis of AV, technical approaches to analytically handle all possible profiles didn't exist. Secondly, the closely related work in economics based on L. Hurwicz's mechanism design, used to analyze systems based on the intensity of preferences and incentives, was still

in its infancy. So, how does one analyze the problem? One way is to impose sufficiently strong restrictions on the system so that a solvable but useful problem emerges. With a clever, insightful approach, Brams and Fishburn restricted attention to a particular subset of situations, and showed that here, AV is optimal. But, as we all know, the best solution in a constrained setting need not have anything to do with the best solution in a global setting. Even if AV is useful in certain, constrained settings, it is not the global optimum. The usual AV restrictions impose a type of monotonicity on the system, and here BFM's stories hold. Once the restrictions are lifted, the answers change.

What are the restricted circumstances where AV may be reasonable? The answer is more general than stated in BFM (see [1]), but we will use their words. BFM point out that "*..AV requires no [assumption about the ability of the voters to rank candidates] and, consequently, it may be well suited to large-scale elections with many unsophisticated voters.*" AV works here because we use the dichotomous voter assumption that the unsophisticated voters can only group the candidates into two classes. Here, we need a voting system to permit you to vote yes for the good guys, and no for the others. This feature defines AV. Any other system forces the unsophisticated voter to make distinctions, and the working assumption is that he can't. Consequently, with this strong assumption, one can justify BFM's fear that determinate systems might motivate the, now, "sophisticated" unsophisticated voter to try to manipulate the system. Notice, these are the kind of arguments and scenarios put forth by BFM, and we suspect that in any setting closely approximated by the unsophisticated voter assumption, BFM may be correct. (For instance, if *all* voters adopt *only* the mean utility strategy, then we are back in the dichotomous setting; if they don't, then we may not be.) But once you relax these strict assumptions, the answers change. If you *can* rank candidates, AV now becomes the method that forces you to make distinctions. (Should I vote for my second ranked candidate? What is my mean utility?) Gillman's very real fears are not admitted in this monotonic, restrictive world.

Given all of this, the name "AV" is misleading. Using BFM's argument, a more accurate title for AV is the **Unsophisticated Voter System** (UVS). It may make good sense to use the UVS, or AV, for a society newly experimenting with voting and democracy. But, for any organization where the voters understand distinctions, the UVS is inappropriate. In particular, UVS should *not* be used in professional societies, the Math Association, nor any other society where the members already

are experienced voters. In our view, this includes all "Western" as well as many "Third World" countries. Obviously, we don't share the BFM's cynical views; we believe that most citizens can rank order a number of candidates. Anyway, if we are asked to accept that voters can compute their "mean utility", and BFM assure us they can, then surely the voters are sophisticated enough to perform the simpler task of just ranking the candidates.

Roulette and games of chance

BFM assert that they *"..find no evidence ..that, because all social orderings are theoretically possible under AV, they may well occur."* Compare their argument to the gambling game of roulette. Before the wheel is spun, all outcomes are possible, but after the wheel stops, only one can occur. Some outcomes are favorable, some are not. The fact that each time the wheel is spun we got only one result is not evidence that the other outcomes could not have occurred. The same is true with the roulette wheel of AV; after all of the ballots are marked, there is only one outcome. Some outcomes are favorable, some are not. Because each election yielded only one result - no matter what - is not evidence that the other outcomes could not have occurred. BFM's argument and conclusions are flawed. Continuing this analogy, the objective of an election is to determine a group's choice, not to play games of chance with an organization's future. Because of AV's large region of indeterminacy, AV seems to do the latter.

Actually, BFM's empirical examples prove just the opposite of what BFM claim. As they admit, both of their profiles admit *indeterminacy* for AV. So, with only simple changes in how these same voters decided to *sincerely* mark their ballots, many different outcomes could occur. We leave it as an easy exercise for the reader to compute some of these changes. You will discover that these modifications in voter decisions are modest, particularly when compared to the drastic changes and assumptions required in BFM's scenarios attempting to discredit BC.

3. STRATEGIC MANIPULATION AND SENSITIVITY

Some of BFM's more interesting observations concern strategic behavior. As BFM admit, they did not attempt *"..a formal analysis of AV's strategic vulnerability vis-a-vis ranking systems, [but] a number of examples as well as computer simulations suggest that AV is far more resistant."* A formal analysis is

carried out in [10], and most of the conclusions directly contradict the conjectures of BFM. We give an intuitive interpretation of these results, and suggest that the interested reader check [10] for details.

To start, plurality voting provides you with a certain number of strategies to manipulate the outcome. Next consider the system that requires you to vote for your two top ranked candidates; it presents you with a different set of strategies. With AV, you are free to choose which system you want to use, so both sets of strategies are available. Intuition suggests that this makes AV more susceptible to being manipulated than either of the original two systems - you have more strategic options. In [10], this is shown to be true. In fact, it follows from [10] that *AV is more susceptible to being manipulated by small groups than any ranking system.* This supports common sense.

There is an exception; in the highly specialized world of unsophisticated voters, UVS does quite well. As true for many of BFM's assertions, this reflects the restrictive, monotonic nature of their assumptions. But, different assumptions yield different results. Indeed, it is proved in [10] that "*..with the appropriate assumptions, with the correctly constructed scenario,* ***any*** *system can be justified as being strategically the best.*" (For related statements also discussing the restrictive monotonic assumptions of AV, see Niemi [5] and the responses referenced in BFM.)

The conclusions of [10] are based on the following approach. Start with the sincere way the voters mark the ballots. Next, count the number of profiles where a small group of voters, by changing their profiles and/or how they choose to mark their ballots, can convert the outcome into a personally more preferable one. The more profiles that can be successfully manipulated in this manner, the more susceptible the system. This is analytically computed in [10]; approximate computer experiments and simulations are not necessary. The results in [10] prove that *AV is more susceptible to manipulation than any ranking system, including the plurality vote and BC.* In fact, AV is more susceptible to manipulation than many multiple systems. This contradicts BFM's conjectures.

"*Sensitivity*" of a system is a related concept. Define it as being where only small changes in how the ballots are marked alter the outcome. (This differs from "manipulation" because the new outcome may not be personally more favorable.) To compare the sensitivity of systems, use the same approach; the more profiles where changes of this kind alter the outcome, the more sensitive the system.

Although this concept is not discussed in [10], an even easier computation (using the techniques of [10]) prove that *AV is more sensitive than any ranked system, and it is more sensitive than many other multiple systems.* This conclusion again supports common sense; AV involves so many variables that we accurately may suspect AV to be a highly volatile method. This contradicts BFM.

Return to the manipulability of AV. What are the strategies? They, of course, depend upon what it takes to "favorably" change the outcome. By carefully examining the proof in [10], several "non-monotonic" situations emerge that are not admitted by the implicit restrictions imposed in BFM. For instance, there are situations where a voter's three top ranked candidates are A>B>C, but, to keep B from beating A, the voter votes approval only for A and C. This behavior, related to Gillman's fears, cannot occur under the restrictive assumptions of BFM.

AV admits other, more familiar "strategic" situations. For instance, suppose, as a candidate, I also approve of another candidate. Most likely I will vote only for myself rather than the two of us. Am I voting legitimately; of course. Am I trying to manipulate the system; absolutely. This strategy was used when AV was used in an AV election in Pennsylvania. As Senator Terry Sanford (when he was at Duke University) noted, *"The great weakness, it seemed to me, was that most voters .. are inclined to cheat a little and 'single shot' if it suits their purposes, which it generally does. I was present for the Pennsylvania straw vote, helped explain it, and was not surprised when very few who voted for [candidate A] voted for anyone else, although surely there were other acceptable candidates."* [6]. Were these voters voting legitimately; of course. Were they trying to manipulate the outcome; absolutely. To call such a vote "sincere" is a game of semantics, not of fact.

This kind of strategy is related to BFM's comment disputing "..*S&VN's argument that the choice of a tallying method is a manipulative tool..*". Of course it is a manipulative tool, but in S&VN we neither attempted to develop this argument nor did we claim full credit for the insight. One of the referees, an author of BFM, asked to have [2] referenced in S&VN. We agreed. To show how the ideas of S&VN are related to [2], we noted that, through indeterminacy, the central theme of [2] extends to all multiple methods including AV. Namely, AV not only admits more manipulative strategies, but it also sanctions some of them. We made the extension; the original insight is due to Brams and Fishburn.

BFM construct scenarios with their election examples to demonstrate what they view as being a terrible, strategic consequences of using BC. But, as we noted, with the identical profiles, AV already admits the outcomes BFM deplore, and many others, *without even requiring the voters to manipulate the system.* With fewer voters sincerely changing the number of candidates they approve of, all of BFM's fears are quickly realized with AV. If you accept BFM's argument about BC - an argument that requires extreme numbers of voters all to be manipulative in a specific manner - then you must be very concerned about the same, but more likely danger (requiring only sincere voters) created by AV's indeterminacy.

By using Theorem 2, this relationship between AV and BC generalizes. Any change of profile that alters the BC outcome already is an admissible AV outcome with a more modest change in the profile. (This is because the BC outcome *always* is in the interior of the convex hull of the admissible AV outcomes.) Thus, if you are measuring sensitivity, manipulation, etc., anything that goes wrong with BC already is an AV fault, but with AV it could occur sooner, easier, and with less extreme scenarios. Indeed, reexamine BFM's argument. It shows that to get an unfavorable BC result, you may need to hypothesize that most voters have a specific Machiavellian attitude, yet the same negative traits already could arise as an AV outcome with sincere voters. AV's indeterminacy is costly. These statements, supported by analytic arguments, contradict the conjectures of BFM.

4. Conclusion

By continuing to exploit the technical structures of multiple systems, described in the final paragraphs of S&VN, it would be easy to demonstrate still more indeterminacy flaws of AV. But, we restricted attention to the concerns raised by BFM, and, even here, the message is clear. First, our answers to BFM's comments are based on analytic arguments over *all* profiles, and these responses contradict most of BFM's claims and conjectures based on examples, scenarios, and computer experiments. As the analysis proves, AV is highly susceptible to indeterminacy, to violating the voters' true wishes, to being manipulated, and AV is more sensitive to small changes in voter opinion than any ranked system. These negatives are serious enough to wonder whether AV should be recalled from the public market until it can be more carefully reexamined. Indeed, it is the responsibility of an advocate of AV not just to promote the positive aspects of AV,

but also to analyze the tradeoffs so we can examine the bottom line. If such an analysis shows that the tradeoffs of AV create a surplus, then we (S&VN) would be pleased (if only because it would involve new structures). But, at the moment, no such analysis exists, and what we know and can derive from S&VN is not encouraging for AV. Indeed, what we know reaffirms our earlier suspicions; the technical properties of AV "*..appear to be sufficiently bad to disqualify approval voting as a viable reform alternative.*"

What about BFM's concern about the "*rigidity of a deterministic system*?" There may be settings where this is a valid concern. But, because AV fares so poorly against other multiple voting systems, we have options other than accepting the flaws of AV. Namely, to preserve certain AV properties that deterministic systems don't have, maybe we should consider other multiple methods. Other multiple systems have many of the suggested merits of AV, but without the same, drastic negatives. If the minuses of such a system are not so severe, it *might* be easier to show that appropriate tradeoffs justify using it. As just one of many possible examples, consider the *Responsive Voting System* (RVS). In the RVS, you rank the candidates in the usual fashion, except now, at any level, you can be indifferent with two candidates. Secondly, you can stop marking the ballot once you run out of approved candidates. This imposes no strain on the voters; an unsophisticated or manipulating voter can stop at the first level. The tabulation is with computers using the BC. When a ballot shows indifference at a level, the number of points assigned to each candidate is the obvious average of what they would receive under BC. For instance, with four candidates A, B, C, D, a ballot marked A>B>C>D would assign A 3 points, B 2 points, and C 1 point; the ballot A=B>C=D would provide A and B each with 2 1/2 points, and C and D each with 1/2 points; and the ballot A>B would assign A 3 points and B 2 points. Not only does RVS have many of the same responsive properties as claimed for AV, but RVS can be used with a mixed group consisting of unsophisticated and sophisticated voters without forcing anyone to compute their "mean utility". RVS does have manipulability, indeterminacy, and sensitivity problems, but they are nowhere near as severe as for AV. (The truncated Borda Count (S&VN) is even less susceptible, and, probably is a much better choice.) In other words, the methodology now exists to analyze all systems, both multiple and simple, without needing to impose restrictive assumptions on the profiles and without restricting our comparison to only one or two other systems. No longer need we base our arguments primarily

against just the PV; a system few, if any, support. We can be more adventurous, complete, and accurate in our search for reform alternatives.

Finally, although we have increasing doubts about using AV, as we indicated earlier we continue to view Fishburn's technical prowess developing the properties of AV and the AV results of Brams and Fishburn as constituting an important contribution. AV is a clever idea discovered by R. Weber, by Brams and Fishburn, and by others that was based on a nice dream. The supporting body of literature raised new issues, created new technical approaches, and stimulated a dialogue. We hope it might be possible to prove that AV has a positive balance outside of approximate UVS situations, but we doubt it. We have a growing suspicion that AV will turn out to be an attractive idea that just didn't pan out. (This doesn't mean that AV still won't be used.)

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