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by

Steven J. Brams,
Peter C. Fishburn
and
Samuel Merrill, III

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NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
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Steven J. Brams
Department of Politics
New York University
New York, NY 10003

Peter C. Fishburn
AT&T Bell Laboratories
Murray Hill, NJ 07974

Samuel Merrill, III
Department of Mathematics
Wilkes College
Wilkes-Barre, PA 18766

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Abstract

Saari and Van Newenhizen (1987) misinterpret their findings about the indeterminacy of voting systems: far from being a vice, indeterminacy is a virtue in allowing voters to be more responsive to, and robbing them of the incentive to misrepresent, their preferences. The responsiveness of approval voting, in particular, means that the cardinal utilities that voters have for candidates can be reflected in voters' strategy choices, which ultimately translate into better social choices. Empirical data from a recent election support these claims, showing, among other things, that the Borda system, a determinate voting system favored by Saari and Van Newenhizen, is highly vulnerable to manipulation by strategic voters.

1. Introduction

Saari and Van Newenhizen (1987; hereafter S & VN) define a voting system to be decisive if a given set of voters' profiles (that is, individual rankings) uniquely determines a social ranking. They prove, correctly, that approval voting and other multiple voting systems not only fail to be decisive in this sense but also that they can be completely indeterminate: any social ordering is attainable, depending on how voters "tally their ballots." (Gillman, 1987, provides an elegant 4-candidate, 12-voter example that illustrates how approval voting can produce all possible candidate orderings.) "Tallying their ballots" refers to how many candidates are given approval votes by voters, moving down their preference orders with no skipping.

S & VN's mathematical results illuminate and extend previous works of Saari (references in S & VN) and others (Fishburn and Gehrlein, 1976; Fishburn, 1981; Gehrlein, Gopinath, Lagarias, and Fishburn, 1982) that use linear algebra to analyze certain voting procedures. What we question is their extra-mathematical interpretations, especially with regard to approval voting (AV). We take this opportunity to offer a rather different perspective: the indeterminacy of AV, which S & VN view with alarm, is in fact salutary because it reflects the ability of AV to be responsive to the desires of the electorate.

Thus, instead of claiming, as S & VN do, that voting systems should avoid indeterminacy, we believe that the flexibility that AV introduces into the voting process--by allowing voters better to register their preferences--is a signal virtue. Why should ordinal information from individual voters uniquely determine the social ranking? Why should not a voter, who ranks five candidates in the order ABCDE, vote differently under AV if he values A highly but not the other four than if he finds A, B, C, and D acceptable but despises E? Of course, the social ranking will be different if many voters make such distinctions, which is exactly as it should be.

This example is a special case of the effect of cardinal, as opposed to ordinal, preferences on the decision of how many candidates to vote for under AV. Whatever criterion the voter uses, it seems rational for his approval ballot to indicate considerations of not just ordinal but also cardinal information. More than a mere "tally of ballots," the number of candidates chosen by an approval voter is a meaningful reflection of the voter's decision calculus, which we shall briefly describe later.

In this light, S & VN's argument that the choice of a tallying method is a manipulative tools loses its force. A voter cannot achieve a better result by altering his optimal strategy. To be sure, voters could achieve a different result by voting differently, but they need not do so unless they prefer the alternative outcome. Doubtless, some voters under AV may choose the candidates whom they vote for capriciously. Such choices indeed may change the outcome. But this is a practical, not a theoretical, pitfall to which any voting system is susceptible.

We shall expand on these ideas in section 3. But first we shall offer three secondary arguments, the last of which, relating to the comparison of voting systems based on their strategic manipulability, is the most substantial. It brings us naturally to our main contention about the responsiveness of AV, which we develop both in terms of S & VN's motivating example and some recent empirical data on the (actual) use of plurality voting (PV) and the (hypothetical) use of AV in the same election.

2. Secondary Arguments

I. Dichotomous Preferences. S & VN may give some readers the impression that previous analyses of AV, and especially comparisons to other voting procedures, rest on the assumption of dichotomous voter preferences, which arise when each voter divides the candidates into two subsets, one relatively preferred to the other, with approximate indifference among candidates within each subset. We do consider dichotomous preferences, of course, but the analyses in Brams and Fishburn (1983) and other references noted therein go well beyond the highly specialized dichotomous case.

II. Ranking Assumption. Because S & VN focus on situations wherein

they presume that voters can rank candidates, we should mention that AV requires no such assumption and, consequently, it may be well suited to large-scale elections with many unsophisticated voters. All that AV asks is that a voter distinguish candidates, if any, whom he finds best suited for the contested office. Academics may lament the inability of the person-in-the-street readily to order a number of candidates, but our democracy encourages citizen participation, and AV would seem to be a simple procedure that facilitates such participation.

III. Strategic Manipulability. We contend that the rigidity of a determinate system, such as PV or the Borda system (BS), is one reason why voters often have a strong incentive to try to manipulate the outcome by strategically misrepresenting their preferences. Although the vulnerability of these systems to strategic misrepresentation has been well documented (Straffin, 1980; Riker, 1982), vulnerability has only recently been tied to restrictions placed on preference expression (Nitzan, Paraush, and Lampert, 1980; Nitzan, 1985); it has not been linked to the determinateness of voting systems.

Under indeterminate systems like AV, the individual voter is the sole arbiter of his choice of strategies. Not being straitjacketed by the strictures of a determinate system, he can better express his preferences than under a nonranking system like PV.

True, a determinate ranking system like BS allows the voter fully to express strict preferences, if he has them. This determinacy becomes a chimera, however, if voters, seeking to escape its strictures, try to subvert them by voting strategically. Unfortunately, S & VN ignore the reasons that voting systems might be subject to manipulation. But since the strategic

properties of a voting system seem intimately related to its determinacy, we cannot ignore these properties in a study of the effects of indeterminacy on social choices.

Because all voting systems are vulnerable to manipulation (Gibbard, 1973; Satterthwaite, 1975; Moulin, 1983), the appropriate question is which systems are least so. We know that AV is more sincere and strategyproof than other nonranked voting systems (Brams and Fishburn, 1983). Additionally, in an exchange with Niemi (Brams and Fishburn, 1985; Niemi, 1985) about claims found in Niemi (1984), AV was shown to surpass PV on several different social-choice criteria in games wherein voters choose sophisticated strategies.

Although we have not attempted a formal analysis of AV's strategic vulnerability vis-à-vis ranking systems, a number of examples as well as computer simulations suggest that AV is far more resistant. The extent to which the indeterminacy of a voting system may protect against manipulability appears to be a fruitful area for further study. Yes, anything is possible under AV and under other indeterminate systems, but the voters' own (quite proper) optimality calculations surely will tend to counter the bizarre cases that S & VN conjure up, as we shall now argue in greater detail.

3. The Responsiveness Issue

To address the main issue of indeterminacy versus responsiveness, we assume henceforth that voters can rank candidates unambiguously. We consider S & VN's initial 15-voter example:

6 voters have preference order ACB;

5 voters have preference order BCA;

4 voters have preference order CBA.

Thus, as S & VN note, C is the Condorcet candidate, who can beat both A and B in pairwise contests. A wins under PV, and any one of A, B, or C can win under AV, depending on who votes for their second as well as first choice. Hence, AV is indeterminate for the given preference profile; later in their paper S & VN argue that such profiles are not at all uncommon. Although the likelihood of indeterminate profiles for AV is not at issue here, we note later some empirical evidence relevant to this matter.

What is at issue here is S & VN's opinion that AV's indeterminacy is an unmitigated evil. Early in their paper they suggest that AV "has several disturbing features that seem to make it worse than even the plurality voting system," adding that it has properties "sufficiently bad to disqualify [it] as a viable reform alternative." They then make clear that the bad properties, insofar as their paper is concerned, are just one: indeterminacy. A little later they acknowledge that one argument for AV and other "multiple systems is that they account for the intensity of preference or distaste for certain candidates."

In fact, one of our principal contentions all along has been that AV does take account of intensities of likes and dislikes far better than does its chief contender, PV. AV does this, moreover, in just about the simplest way possible without getting voters involved in exercises they may find difficult, or whose results they may find puzzling. Indeed, we regard AV's indeterminacy as desirable precisely because it demonstrates that AV responds positively to distinctions voters make among candidates that ordinal preference rankings do not mirror.

For example, consider three scenarios for the foregoing 15-voter profile that give different winners under AV:

Scenario 1. Voters passionately like their first choices and dislike the other two, having only a slight preference between their last choices. As under PV, each person votes only for his first choice, so A wins.

Scenario 2. The 11 A-first and B-first voters are the same as in scenario 1, but the four C-first voters like B almost as much as they like C, and they detest A. Hence, they vote for both C and B, and B wins.

Scenario 3. The 11 A-first and B-first voters all like C nearly as much as their first choice and much more than their third choice, so all vote for C as well as their first choice. Thus, C emerges as the winner with unanimous support from the electorate.

We submit that the winner under each scenario arguably is the best candidate for that scenario in terms of the voters' desires. While we doubt that S & VN would condemn the social choices in these three different scenarios, they do wonder whether this kind of responsiveness is "worth the accompanying cost of indeterminacy." Our response, as before, is that their indeterminacy is not a vice but instead a surpassing virtue of AV.

Although the preceding scenarios may be overdrawn and involve quasi-dichotomous preferences, they do illustrate our argument. For more ordinary worlds, with many more voters and less passion among them, analyses under many sets of starting assumptions (Brams and Fishburn, 1983; Merrill, 1988; and references in each) suggest that a voter under AV will do as well as possible from a purely self-interested perspective by voting for the candidates whose utilities are above his mean utility over the set of candidates. This rule, of course, is what the scenarios presume, and the analyses indicate that if this rule more or less obtains in practice, then voters will elect "good" candidates. In other words, self-interest maximization by voters leads to socially desirable outcomes.

A corollary of the mean-utility principle is that in an election among three candidates, about one-half of the voters who vote for either one or two candidates will vote for exactly one, and about one-half will vote for exactly two, provided that most voters' evaluations of their second choices do not generally lie toward one extreme, as in scenario 1. We now have a bit of new data that bears on this, as well as on the matter of the likelihood of indeterminacy under AV.

The data come from one of the first major field tests of AV, conducted by The Institute of Management Sciences (TIMS) in conjunction with its 1985 elections. (For comparable empirical data leading to similar results, see Felsenthal, Maoz, and Rapoport, 1986; and Rapoport, Felsenthal, and Maoz, 1987.) Fishburn and Little (1987) present the experiment in detail. Here we note only a few features of the two three-candidate elections studied that related to S & VN's claims.

Members of TIMS received official plurality ballots for their 1985 elections along with experimental ballots that asked them to rank candidates and mark approvals. A total of 1,851 members returned the official ballots, and 1,511 of these (82 percent) also returned the experimental ballots with both rankings and approvals noted thereon. The data we present here involve only the experimental ballots for the two three-candidate elections, denoted by E1 and E2.

The less-than-1,511 totals that we report on below reflect two facts: some voters who ranked all three candidates in E1 did not do so in E2 (or vice versa); and some voters in each election either approved of no candidate or (more likely) approved of all three.

Table 1 shows the numbers of voters who cast an approval vote for

Table 1 about here

exactly one or exactly two candidates. The E1 data indicate almost an even split between these strategies, whereas the E2 data show about a 5-to-3 edge for one approval versus two approvals. In either case there was not an overwhelming propensity to vote for only one candidate on an approval ballot, as the official plurality ballot instructed voters they must do.

Table 2 shows the frequencies of complete rankings (no ties, which

Table 2 about here

were permitted but rare) for the three candidates in each election. With regard to indeterminacy, the only candidate in either election that could not have won under AV was A in election E1.

The official plurality winner in each election was C, with B a close second. The ratios of official votes for C-versus-B, and the first-place C/B ratios from the partial data of Table 2, are:

	Official (C/B)	Table 2 (C/B)
E1	835/827 = 1.01	663/634 = 1.05
E2	597/552 = 1.08	456/422 = 1.08

Hence, the partial data, which give the same ranking (CBA) as the official plurality data on the basis of first choices, are fairly representative of the entire responding electorate.

The rankings of the voters help considerably in discovering if there is a Condorcet candidate. In both elections, each of B and C had a significant majority over A, so either B or C is a Condorcet candidate. However, when we consider the totality of data, in both E1 and E2 the majority com-

TABLE 1

SINGLE AND DOUBLE APPROVALS FOR TWO THREE-CANDIATE ELECTIONS (E1 AND E2)

		Election E1	Election E2
Number of Approvals	1	660 (48%)	720 (63%)
	2	706 (52%)	418 (37%)
Totals		1366 (100%)	1138 (100%)

TABLE 2

FREQUENCIES OF PREFERENCE RANKINGS

		Preference Ranking ABC	ACB	BAC	BCA	CAB	CBA	Totals
Election	E1	73	66	294	340	168	495	1436
	E2	162	129	181	241	172	284	1169

parison between B and C was too close to call--essentially a tie (Fishburn and Little, 1987). With respect to the partial data of Table 2, C has a 729-to-707 majority over B in E1, and C has a 585-to-584 majority over B in E2.

Whom did AV elect? In one election it agreed with PV, but in the other it did not. The switch occurred in E1, in which B had 1,038 approvals and C had 908 (ratio = 1.14). (A had 417 approvals.) The switch arose because (1) more voters who ranked A first approved of B than C (36 percent to 23 percent), and (2) proportionately more of C's supporters approved of B than did B's of C. Thus, the very close official ratio (C/B = 1.01) and the dead-heat majority estimate were considerably widened by AV (B/C = 1.14).

In E2, C had 768 approvals to 713 for B, for a C/B ratio of 1.08, which is about the same as the official plurality ratio. (A had 558 approvals.) The approval counts, incidentally, include some voters not accounted for in Table 2--namely, approval voters who did not rank all candidates.

Finally, we note the likely results of each election under BS, which S & VN endorse. Using only the data of Table 2, with Borda points of 2, 1, and 0 for first, second, and third places, in E1 B has a Borda score of 1,836, C a score of 1,732. Thus, BS and AV agree for E1, and both disagree with PV. The B/C ratios are 0.99 for PV, 1.06 for BS, and 1.14 for AV.

In E2 the Borda scores from the data of Table 2 are 1,290 for B and 1,282 for C, giving a very slight edge to B. Because these scores are so close, it seems fair to say that the election is indecisive, similar to the B-to-C majority comparison (484 to 485 from Table 2). As we noted earlier, the PV and AV counts both favor C over B by a ratio of about 1.08.

Both BS and AV show up the extreme closeness of this race, with the C-to-B ratio perhaps suggesting that C should receive the nod. Recognizing this closeness beforehand, however, individual voters under BS effectively could misrepresent their preferences by ranking their favorite (say, B) first and his close competitor (C) last, even though C might be their second choice.

If all such voters followed this strategy, then not only would the order of finish of B and C under BS have been reversed, but, astoundingly, A would have shot up from last place to win an easy victory! If, in addition, voters in groups ABC and ACB had separated B and C by dropping A to second choice, a virtual tie would ensue, as the BS totals for E2 derived from Table 2 indicate (see Table 3). Although in practice not all voters would vote

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Table 3 about here

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strategically, this example suggests the extreme volatility of results under BS.

On the other hand, under AV voters whose top choices were either B or C simply might have bullet-voted for one of them. This is a sincere choice because it does not involve reversing the ranking of candidates, as under BS (for example, switching from a sincere BCA to an insincere BAC strategy), or skipping preferred candidates. Indeed, assuming that all voters in E2 had shifted their strategies, if necessary, so as to vote for one of B and C but not both, the order of finish would switch B and C (see Table 4), but the election again remains very close--essentially indecisive,

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Table 4 about here

TABLE 3

BORDA SCORES AND SOCIAL ORDERINGS

	Candidate			
	A	B	C	Social ordering
Sincere voting	935	1290	1282	BCA
Manipulation by BCA and CBA groups	1460	1006	1041	ACB
Manipulation also be ABC and ACB groups	1169	1168	1170	Virtual Tie

TABLE 4

APPROVAL VOTING TOTALS AND SOCIAL ORDERINGS

	Candidate			
	A	B	C	Social ordering
Sincere voting	558	713	768	CBA
Strategic voting	558	638	632	BCA

echoing the earlier nonstrategic comparisons--and a far cry from the radical shift that strategic voting under BS would have produced. In general, the indeterminate AV is less vulnerable to manipulation than the determinate BS in an election, such as the TIMS race, in which voters might anticipate a close contest and therefore have good reason to vote strategically.

4. Conclusions

Indeterminacy, like inconsistency, may sound disconcerting, even loathesome; reinterpreted in terms of responsiveness, however, it is a desirable property of a voting system--at least in its AV incarnation. We find no evidence to support S & VN's view that, because all social orderings are theoretically possible under AV, they may well occur. Many analytic studies under a variety of assumptions, and empirical studies in very different kinds of elections--including that briefly discussed here--show their view to be without foundation.

Depending on how voters rank candidates, or what cardinal utilities they attach to them, voters under AV can, and apparently do, choose the size of their approval sets rationally, not at random. Their presumed rational (nonrandom) choices both drastically reduce the alleged indecisiveness of AV and do so in a proper way.

S & VN limited their analysis to certain possibility (existence) results, without inquiring into the probability that these results ever would occur, especially in light of what rational voters would do under different circumstances. With commendable detachment, they averred that "the decision to adopt one system over another involves other issues," but they then offered a wholesale condemnation of AV.

We think S & VN have done a service in bringing to our attention the concept of the determinacy of a voting system. We fear that their fixation on this concept has blinded them to trade-offs with other criteria, particularly how some determinate systems beg to be jimmied because of their inflexibility and how voter optimality calculations, as least under AV, tend to render indeterminacy quite harmless despite the plethora of possible outcomes that--in theory, but almost surely not in practice--may occur.

We would like to see some of these trade-offs formalized, and perhaps S & VN will help us in this endeavor. Until then, we hope they will follow their own good advice and eschew the blanket indictment of a system such as AV on the basis of an analysis of only one feature of a voting system, which seems to us misinterpreted and intimately tied to other features that also require evaluation if we are to reach balanced conclusions.

There is perhaps a larger, more philosophical lesson in all this. We take the normative position that a voting system should afford voters considerable freedom to structure their own choices on the basis of their preferences, in part because this freedom tends to rob them of the incentive to be dishonest or manipulative in selecting a voting strategy. There is considerable empirical evidence, moreover, that voters <u>are</u> responsible, and <u>do</u> make sensible if not always optimal individual choices. The social choices that flow from these individual choices under AV, our studies indicate, do not often betraw the freedom that we entrust to voters.

To be sure, freedom may have a price in permitting a possibly dreadful outcome. Yet a growing body of studies flatly contradicts the proposition that AV seriously will jeopardize Condorcet winners, much less select Condorcet losers. Quite the contrary: it is AV, not PV, that compares favorably

with ranking systems on the basis of Condorcet and other important criteria of the quality of social choices. And it is AV, not BS, that comes out at or near the top on non-manipulability.

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