And the loser is... Plurality Voting

Jean-François Laslier [Draft of April 26, 2011]

Abstract This paper reports on a vote for choosing the best voting rules that was organized among the participants of the Voting Procedures workshop in July, 2010. Among 18 voting rules, Approval Voting won the contest, and Plurality Voting received no support at all.

1 Introduction

Experts have different opinions as to which is the best voting procedure. The Leverhulme Trust sponsored 2010 *Voting Power in Practice* workshop, held at the Chateau du Baffy, Normandy, from 30 July to 2 August 2010, was organized for the purpose of discussing this matter. Participants of the workshop were specialists in voting procedures and, during the wrap-up session at the end of the workshop, it was decided to organize a vote among the participants to elect "the best voting procedure". The present paper reports on this vote. It contains in the Appendix statements by some of the voters/participants about this vote and voting rules in general.

2 The vote

Previous discussion had shown that different voting rules might be advisable under different circumstances, so that a more concrete problem than "What is the best voting rule" should be tackled. The question for the vote was: "What is the best voting rule for your town to use to elect the mayor?"

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Even with this phrasing, it was realized afterwards that not all participants had exactly the same thing in mind. In particular, some of them were thinking of a large electorate and some were rather thinking of a committee (the city council) as the electorate. This can be inferred from the participants' comments in the Appendix and is clearly a weakness of this "experiment."

Of course, another interesting feature of this vote is the fact that it was a vote on voting rules *by voting theorists*. So the participants arrived with quite a heavy background of personal knowledge and ideas. But the way the vote was improvised was such that no one had much time to think things over, discuss and coordinate with others, or calculate. Moreover, no candidates were clear common knowledge front-runners, and the final result was apparently not anticipated by most voters.

The possibilities of strategic manipulation were thus quite limited and one can indeed see from the comments that most of these approval votes should be interpreted as the expression of sincere individual opinions. As one referee pointed out: this vote may be the last "naive" vote on voting rules. This adds a particular significance to its result, and also suggests that the experiment should be done again, now that the results are known.[1]

2.1 Candidates: the voting rules in question

The set of "candidates," that is the list of considered voting rules was rather informally decided: participants just wrote on the paper board voting rules to be voted upon. Eighteen voting rules were nominated, the definitions of which can be found in the Appendix 2.

Some rules should really be considered as possible ways to organize elections and will, in usual circumstances, provide indeed a unique winner. Others will often yield not a single winner but a set of possible winners, among which the final choice has to be made by one means or another. The list contains several Condorcet-consistent rules which agree on a unique outcome when the Condorcet winner exists[2] but which differ when there is no Condorcet winner and, in that case, often yield several winners. For instance the Uncovered set is a singleton only if there is a Condorcet winner[3], the Copeland winner is always in the Uncovered set and the Uncovered set is always included in the Top Cycle. The voting rules also differ as to their informational basis.

[1] My guess, based on the theoretical analysis of strategic voting under Approval Voting, is that the result would not be different.

[2] The existence of several Condorcet winners simultaneously is a rare phenomenon.

[3] No randomization scheme was considered. In particular the optimal solutions to the Condorcet paradox studied by Laffond, Laslier and LeBreton (1993) and Dutta and Laslier (1999) were not on the list of voting procedures.

1. Some of them require very little information: Plurality voting and Majority voting with a runoff simply ask the voter to provide the name of one (or two) candidates. Approval Voting asks the voter to say "yes" or "no" to each candidate.
2. Most rules require that the voter ranks the candidates: this is the classical framework of Arrowian social choice (Arrow 1951). There is no inter-personal comparisons of alternatives, which means that the ballots are not intended to convey interpretation of the kind "candidate *a* is better for voter *i* than for voter *j*". The intra-personal structure is purely ordinal, which means that we may know that *a* is better than *b* for *i*, but we cannot know *how much* better.
3. Finally, some of them allow inter-personal comparisons, with intra-personal comparisons being ordinal (Leximin, Majority Judgement) or cardinal (Range Voting).

Most rules extend the majority principle in the sense that, if there are only two candidates, they select the one preferred by a majority of the voters. Range voting, which maximizes the average evaluation, does not fulfill this principle: indeed, according to classical utilitarianism, if a majority of voters slightly prefer *a* to *b* while a minority strongly prefers *b* to *a*, it may be better to choose *b* than *a*, against the majority principle. Therefore, under Range voting, if voters reflect in their vote this pattern of interpersonal comparisons, the minority candidate *b* may well be elected against *a*. Such is also the case for the "Majority judgement" system (despite its name) which maximizes the median evaluation and for the Leximin, which maximizes the worst evaluation.

2.2 The procedure

2.2.1 The electorate

The 22 voters were the participants of the workshop (one participant abstained). Some of them are advocates of a specific voting rule; for instance, Ken Ritchie and Alessandro Gardini are active in Great Britain in promoting the "Alternative Vote": a system of vote transfers also known as the "Hare" system. Others, like Dan Felsenthal, are advocates of the Condorcet principle and strongly defended this principle during the workshop. But it is fair to say that most of the participants would say that different voting rules have advantages and disadvantages. This might be one of the reasons why no-one objected to the use of Approval voting for this particular vote.

2.2.2 The voting rule

We used Approval voting for this election. Somebody made the suggestion and there was no counter-proposal. In retrospect, this choice was quite natural: this procedure is fast and easy to use even if the number of candidates is large. Asking voters

Number of approvals	0	1	2	3	4	5	6	7	8	9	10	>10	total
Number of ballots	0	2	7	3	5	2	1	1	0	0	1	0	22

Table 1 Number of approved candidates

to rank the 18 candidates was hardly feasible in our case. Approval voting is also advisable when the set of alternatives has been loosely designed and contains very similar candidates[4]. One may nevertheless regret that we lost the occasion to gather, through the vote, more information on the participants' opinions about the different voting rules. Hopefully the next section, where results are presented, will show that we can already learn quite a lot from the analysis of the Approval ballots.

3 The results

3.1 Approval score and other indicators

Voters approved on average 3.55 candidates out of 18, with a distribution provided in Table 1. This figure is not at odds with what has been observed in other circumstances (Laslier and Sanver 2010).

Table 2 provides the scores of the candidates:

Approvals This is the number of voters who approve the candidate.
Approval score This is the percentage of the population who approve the candidate. *Approval Voting* is approved by 15 voters out of 22, that is 68.18%.

Approval Voting is the winner of the election. It is worth noticing that it is the only candidate approved by more than half of the voters[5]. Three candidates received no vote at all: *Fishburn*, *Untrapped Set*, and *Plurality*.

There are actually many different ways to compute scores and other indicators from a set of Approval ballots. Table 3 provides some, which are now defined. The number of voters who approved of both candidates c and c' is called the association of c and c', and is denoted by $as(c,c')$. The number of voters who approved c is denoted by $as(c)$.

Markov score This score is computed as follows. The candidate "present at date t" is denoted $c(t)$. At date t chose at random one voter v. If v approves $c(t)$, keep this candidate for the next date: $c(t+1) = c(t)$. If not choose $c(t+1)$ at random among the candidates that v approves. This defines a Markov chain over candidates whose stationary distribution is the Markov score. For instance a candidate with Markov score .3 is, in the long run of this process, present 30% of the time.

[4] See the cloning-consistency condition (Tideman 1987) and the composition-consistency property (Laffond, Lainé and Laslier 1996).

[5] One voter wrote on his/her ballot "Approval Voting with a runoff." This procedure was not on the list. This ballot was counted as an approbation of Approval voting.

Voting Rule		Approvals	Approving percentage
Approval Voting	`App`	15	68.18
Alternative Vote	`Alt`	10	45.45
Copeland	`Cop`	9	40.91
Kemeny	`Kem`	8	36.36
Two-Round Majority	`2R`	6	27.27
Coombs	`Coo`	6	27.27
Simpson	`Sim`	5	22.73
Majority Judgement	`Bal`	5	22.73
Borda	`Bor`	4	18.18
Black	`Bla`	3	13.64
Range Voting	`RV`	2	9.09
Nanson	`Nan`	2	9.09
Leximin	`Lex`	1	4.54
Top-Cycle	`TC`	1	4.54
Uncovered Set	`UC`	1	4.54
Fishburn		0	0
Untrapped Set		0	0
Plurality		0	0

Table 2 Approval scores

Focus The focus of candidate c is the sum over all candidates k of the fraction of c-voters who also approved k.

$$f(c) = \sum_k \frac{as(c,k)}{as(k)}.$$

The focus measures the ability of a candidate to attract votes from voters who also voted for others.

Centrality This indicator is based on the following Markov chain. The transition probability from c to c' is $as(c,c')/\sum_{c \neq c'} as(c,c')$. The centrality measure is the associated stationary probability. This is a natural measure of centrality in the multi-graph where there is a link between two candidates each time a voter approves them both.

Similarity This indicator is based on the following Markov chain. Given the candidate c, one chooses at random a voter v. If v approves c one replaces c by c' chosen at random among the candidates that v approves. If v does not approve c, one replaces c by c' chosen at random among all the candidates. The similarity measure is the associated stationary probability. This means that, given a candidate c, one looks for a candidate c' which is similar to c in the sense that a voter has approved both.

Satisfaction If v has approved $B(v)$ candidates, count $1/B(v)$ points for each. The total count of candidate c is thus between 0 and the number of voters, and the sum over candidates is the number of voters. (See Kilgour, 2010.)

Dilution This is the average number of candidates approved by the voters who approve a given candidate. Let $as(v,c,k)$ be 1 if voter v approves both c and k,

	Approvals	Markov	focus	central.	simil.	satisf.	dilution
App	15	36.44	12.53	15.65	10.44	5.14	4.07
Alt	10	11.70	9.01	11.90	8.39	2.67	4.50
Cop	9	14.60	6.90	9.86	8.12	3.24	4.22
Kem	8	10.56	6.56	8.16	7.61	2.39	4.00
2R	6	6.91	5.50	7.14	6.86	1.71	4.5
Coo	6	6.28	4.88	7.48	6.89	1.63	4.67
Sim	5	2.82	7.17	8.50	6.52	.99	6.00
Bal	5	2.26	4.69	7.48	6.62	1.05	5.40
Bor	4	4.42	5.08	5.78	6.16	1.24	5.25
Bla	3	.86	5.59	6.12	5.83	.47	7.00
RV	2	1.95	1.60	1.70	5.55	.70	3.50
Nan	2	.29	4.35	5.10	5.49	.24	8.50
Lex	1	.42	1.67	1.36	5.17	.20	5.00
TC	1	.30	1.82	1.70	5.17	.17	6.00
UC	1	.21	2.25	2.04	5.16	.14	7.00

Table 3 Various indicators

	Approvals	Markov	focus	central.	simil.	adjust.	dilution
Approvals	1	.935	.930	.958	.999	.980	-.547
Markov	.935	1	.851	.981	.930	.878	-.285
focus	.930	.851	1	.839	.943	.966	-.511
central	.958	.981	.839	1	.952	.897	-.347
simil.	.999	.930	.943	.952	1	.984	-.546
adjust.	.980	.878	.966	.897	.984	1	-.603
dilution	-.547	-.285	-.511	-.347	-.546	-.603	1

Table 4 Correlations among indicators

and 0 ifnot. Then:

$$dil(c) = \frac{1}{as(c)} \sum_v \sum_k as(v,c,k).$$

Notice that this indicator can be computed with a formula somehow dual to the focus:

$$dil(c) = \sum_k \frac{as(c,k)}{as(c)}.$$

The dilution thus measures to what extent supporters of a candidate also vote for other candidates. It should not be interpreted as an indicator of the strength of the candidate but as a part of the description of the electorate of the candidate: do these voters give exclusive support (low dilution), or do they support many other candidates (high dilution).

These indicators are all highly correlated with the approval score, except for the dilution (see Table 4).

	App	Alt	Cop	Kem	2R	Coo	Sim	Bal	Bor	Bla	RV	Nan	Lex	TC	UC
App	**15**	7	7	4	3	3	4	5	3	3	2	2	1	1	1
Alt	7	**10**	2	3	5	4	3	3	1	3	1	1	1	1	0
Cop	7	2	**9**	4	1	2	3	4	2	1	0	2	0	0	1
Kem	4	3	4	**8**	1	3	3	1	1	1	0	1	0	1	1
2R	3	5	1	1	**6**	3	1	1	2	2	0	1	1	0	0
Coo	3	4	2	3	3	**6**	1	2	1	1	1	1	0	0	0
Sim	4	3	3	3	1	1	**5**	2	2	2	0	2	0	1	1
Bal	5	3	4	1	1	2	2	**5**	1	1	1	1	0	0	0
Bor	3	1	2	1	2	1	2	1	**4**	1	0	2	0	0	1
Bla	3	3	1	1	2	1	2	1	1	**3**	0	1	1	1	0
RV	2	1	0	0	0	1	0	1	0	0	**2**	0	0	0	0
Nan	2	1	2	1	1	1	2	1	2	1	0	**2**	0	0	1
Lex	1	1	0	0	1	0	0	0	0	1	0	0	**1**	0	0
TC	1	1	0	1	0	0	1	0	0	1	0	0	0	**1**	0
UC	1	0	1	1	0	0	1	0	1	0	0	1	0	0	**1**

Table 5 Association matrix

	App	Alt	Cop	Kem	2R	Coo	Sim	Bal	Bor	Bla	RV	Nan	Lex	TC	UC
App	**1**	.7	.78	.5	.5	.5	.8	1	.75	1	1	1	1	1	1
Alt	.47	**1**	.22	.38	.83	.67	.6	.6	.25	1	.5	.5	1	1	0
Cop	.47	.2	**1**	.5	.17	.33	.6	.8	.50	.33	0	1	0	0	1
Kem	.27	.3	.44	**1**	.17	.5	.6	.2	.25	.33	0	.5	0	1	1
2R	.2	.5	.11	.12	**1**	.5	.2	.2	.5	.67	0	.5	1	0	0
Coo	.2	.4	.22	.38	.5	**1**	.2	.4	.25	.33	.5	.5	0	0	0
Sim	.27	.3	.33	.38	.17	.17	**1**	.4	.50	.67	0	1	0	1	1
Bal	.33	.3	.44	.12	.17	.33	.4	**1**	.25	.33	.5	.5	0	0	0
Bor	.20	.1	.22	.12	.33	.17	.4	.2	**1**	.33	0	1	0	0	1
Bla	.20	.3	.11	.12	.33	.17	.4	.2	.25	**1**	0	.5	1	1	0
RV	.13	.1	0	0	0	.17	0	.2	0	0	**1**	0	0	0	0
Nan	.13	.1	.22	.12	.17	.17	.4	.2	.5	.33	0	**1**	0	0	1
Lex	.07	.1	0	0	.17	0	0	0	0	.33	0	0	**1**	0	0
TC	.07	.1	0	.12	0	0	.2	0	0	.33	0	0	0	**1**	0
UC	.07	0	.11	.12	0	0	.2	0	.25	0	0	.5	0	0	**1**

Table 6 Conditional association matrix

3.2 Structure of the set of candidates

Table 5 shows the number of voters who approved each pair of candidates, and Table 6 shows the distribution of these association numbers for each candidate. For instance $7/10 = 70\%$ of the voters who approved the *Alternative Vote* also approved *Approval Voting* while $7/15 = 47\%$ of the voters who approved *Approval Voting* also approved the *Alternative Vote*. It is interesting to note that 83% of the supporters of two-round majority voting also support the *Alternative Vote*, but such is the case of only 22% of the *Copeland* supporters. One may also notice that all the supporters of the *Majority Judgement* are also supporters of *Approval Voting*.

To obtain a more global view, one may compute various distances between candidates. Consider for instance for the similarity index

$$sim(c,c') = \frac{as(c,c')}{as(c)} + \frac{as(c,c')}{as(c')}$$

which ranges from 0 (when the electorates of c and c' are disjoint) to 2 (when they are identical) and define

$$dist(c,c') = 2 - sim(c,c').$$

It turns out that there exists a very good Euclidean representation of the 15 candidates in 3 dimensions, that renders 90% of the sum of square of distances.[6] Figures 1 and 2 are side views of this representation. *Approval Voting* is in the center. The points on the right are rules which are important in the social choice literature: *Uncovered set*, *Copeland*, *Nanson*, *Kemeny*, *Simpson*, even if they are not very practical. *Borda* is in this group, close to *Nanson*. The points on the left contain three practical solutions to the voting problem: *Two-round majority*, the *Alternative Vote*, and *Black*. *Leximin* is not far from this group. *Coombs* and *Majority Judgement* are close one to the other, with *Range Voting* not far. The *Top-cycle* is isolated.

This structure reflects the vote profile since by definition, two voting rules are represented close one to the other when the same voters approved both.

Studying how candidate rules are associated in the voters' ballots, it appears that the winner is receiving votes associated with all the other candidates. *Approval Voting* can be described as a "centrist" candidate in this vote. Even if one can detect some pattern in the vote profile that differentiates votes for more "theoretical" rules from votes for more "practical" rules, the electorate does not appear to be split.

[6] See Appendix 3 for more details.

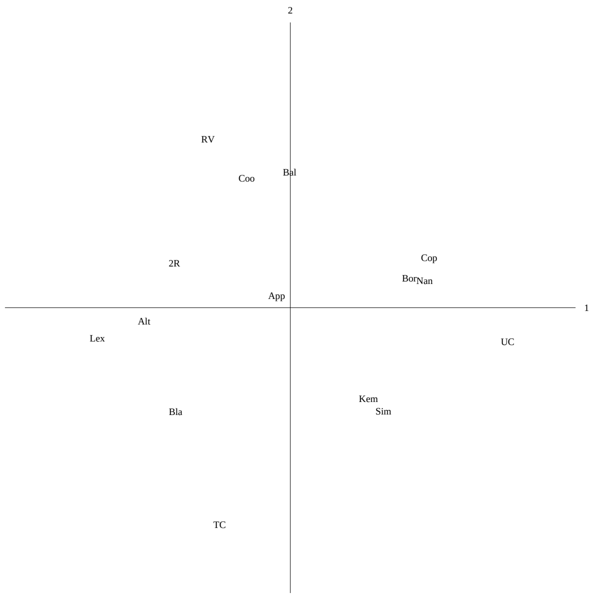

Fig. 1 Axes 1 and 2

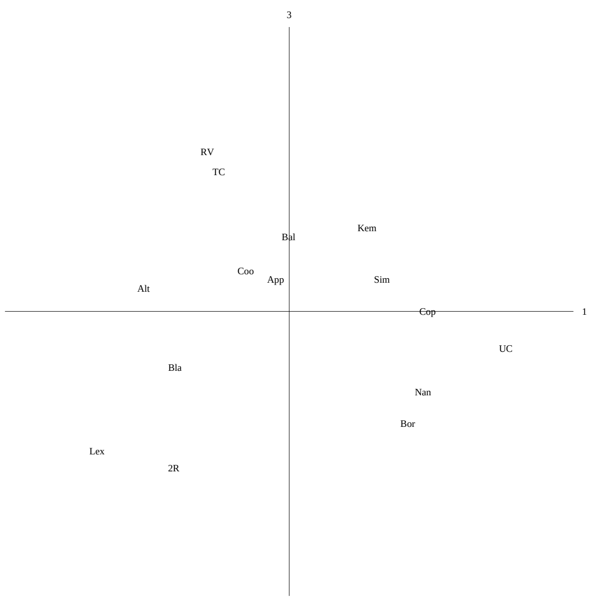

Fig. 2 Axes 1 and 3

4 Conclusion

The analysis of the approval ballots show that *Approval Voting* was a clear winner of this election. This is somehow surprising since *Approval Voting* was not much discussed during the workshop. But this voting rule has already received a lot of attention in the academic literature, and was certainly familiar to the participants.

A striking fact is that *Plurality* rule (First Past the Post) received no approval. The whole ranking of the candidate procedures according to their approval scores seems also robust since alternative ways to count the ballots produce rather similar rankings.

Appendix 1: Contributions of the participants

Participants were ex post invited to write a brief statement that would explain their vote and their view about this "election". One-half of them did it, after having read my own contribution (see below) as an example.

It must be acknowledged that, from a scientific point of view, the experimental protocol was a little loose. The nomination procedure for the set of candidate voting rules was informal and voters did not have all the time required to learn everything about all of them.

> Fuad Aleskerov: "From my point of view our voting for the rules was rather spontaneous. For instance, I know more than 30 rules which can be listed for our voting after reading careful studies, by colleagues, of their properties."

It is also worth remarking that, under Approval Voting, it is not so easy to remember after several days or weeks which candidates you approved out of 18.

> Marc Kilgour: "I really can't remember very well why I voted the way I did. As I recall, the objective was to propose a system to elect the mayor of a town, without any indication of the number of candidates. I think I assumed that there would not be many. I followed the approval strategy of approving everything that seemed to be above average "utility," whatever that would mean. I voted for approval (my actual favorite) and range because they focus on acceptability rather than ranks. I also voted for two or three others that, it seemed to me, were complicated enough to be likely to produce something that would maximize the sum of the utilities but at the same time sophisticated enough to avoid features I don't like such as non-monotonicity. Beyond that, I can't remember much."

This point holds certainly true, as well, for preference-based balloting: it is not so easy to remember how you ranked the whole set of alternatives; it distinguishes these systems from the familiar single-name Plurality and Two-Round Majority voting rules. The set of received contributions, which follow, show well the variety of view points of the experts in the field.

A. Baujard

The question we have been asked implied to apply the rule in a real political context, involving citizens with their own desires, their own intellectual abilities and the differences among them. Choosing the good rule for a democratic mayor election predicates to pay attention to these features rather than to my own preferences among rules. Through the field-experiments we have conducted, I have learnt that most voters are frustrated by voting rules which gives little scope to the expression of their nuanced preferences, which are sometimes tinted by hesitations, indifference, significant differences between strong vs. weak preferences... These nuances have yet a strong impact on results especially in the case of uninominal voting rules, where a clear cut decision is always required to infer individual preferences. In a democracy, I claim that this should question the legitimacy of the winner in such context. I am therefore not convinced by plurality rules, whatever one or two rounds, Condorcet principle, or any uninominal voting rules in general.

Among plurinominal rules, I focus on the importance of simplicity to explain the rule, and to vote. Above all, I paid attention to transparency, meaning a wide understanding of the process of deriving a result from the ballots, and the ability of citizens to take actively part in the process of counting the votes. These desired properties rule out Alternative Vote, Borda rules and Majority Judgement among others.

Many voters in our experiments spontaneously preferred range voting, begging for the ability of giving negative grades, or a wide range of different grades. Even though this would also be my favorite in an ideal world, I regret how range voting depends on differences in the meaning of grades among people, how it is manipulable — which causes strong inequalities among the voting power of different citizens according to their ability to manipulate. This argument, I admit, may be questionable in a reduced city council, but ruling out range voting seemed cautious in the absence of information on its size and composition.

I have eventually given just one approval in the vote on voting rules : one to approval voting. It is because I had the ability of approving other rules that my choice of giving just one vote was truly meaningful.

D. Felsenthal

I adhere to the Condorcet Principle as a normative principle when one must elect one out of three or more candidates. This principle prescribes that should a candidate defeat every other candidate in pairwise comparisons (a Condorcet winner), it must be elected, and should a candidate be defeated by every other candidate in pairwise comparisons (a Condorcet loser), it must not be elected. This principle conveys the fundamental idea that the opinion of the majority should prevail, at least when majority comparisons pinpoint an unambiguous winner and/or an unambiguous loser. The Condorcet Principle takes into account only the ordinal preferences of every voter between any pair of alternatives because attempting to take into account also voters' cardinal preferences (as under the Range Voting procedure) would not only imply that a Condorcet winner may not be elected or, worse, that a Condorcet loser may be elected, but also that inter-personal comparisons of utility are possible and acceptable – which they are not! I rank all the competing procedures for electing one out of m candidates ($m \geq 2$) according to two criteria: the primary criterion is whether a procedure is Condorcet-consistent, and the secondary criterion is whether a procedure is vulnerable to one or more paradoxes which I consider as especially serious. These paradoxes are: not electing a candidate despite the fact that an absolute majority of the voters rank him or her first in their preference ordering, electing a candidate who is a Condorcet Loser or even an Absolute Loser, electing a candidate who is Pareto-dominated, susceptibility to non-monotonicity. According to these criteria I approved only of Kemeny's and Copeland's procedures because they are both Condorcet-consistent and are not susceptible to any of the four paradoxes which I consider especially serious. My rank-order of the 18 competing procedures is as follows:

Kemeny > Copeland > Black > Nanson > Untrapped Set > Fishburn > Uncovered Set > Top Cycle > Simpson > Borda > Coombs > Alternative Vote > 2-round Majority > Plurality > Majority Judgment > Approval Voting > Leximin > Range Voting.

W. V. Gehrlein

In all honesty, I do not remember exactly which of the many possible rules that were listed that I voted for during this impromptu exercise. However, my general convictions were expressed on the ballot that I submitted. The first statement on my ballot was: "In a perfect world I would recommend any Condorcet consistent voting rule". Standard arguments against the implementation of majority rule based voting are too heavily focused on one atypical example of something that could conceivably happen to ignore an 'almost-majority' minority voting bloc with strong preferences. The obvious question is: What is the likelihood that such a scenario would ever actually exist? We all know that such hypothetical voting situations can always be developed to make any voting rule appear to behave very poorly on some criterion. The only practical way out of this dilemma must therefore be based on the likelihoods that voting rules display such bad behavior. In the context of evaluating voting rules to elect the mayor of a city in a typical situation, my assumption from scenarios that I am familiar with would make the possibility negligible that there would ever be more than four candidates. Since there is a very high probability that a Condorcet winner will exist in such cases, why should we not elect that candidate?

The answer to the immediately preceding question is that Condorcet consistent procedures are not always easy to implement with a larger number of candidates, which led to my second statement on the ballot. "In the real world I would recommend (some elimination rules that I do not recall) and Borda Rule". These rules would give a reasonable probability of electing the Condorcet winner, while also being both explainable to and acceptable to the electorate, without any implication that simplicity should be the only criterion for evaluating voting rules. Arguments about the relatively large probability with which some of these voting rules can be manipulated are typically based on the assumption that one group of voters with similar preferences can manipulate the outcome, while all other voters are completely naive to the situation. When it is further assumed that these other voters are aware of such possibilities and that they can react accordingly, the probability that the winner could actually be changed is significantly reduced. However, it is definitely reasonable to conclude from this exercise that plurality rule is not considered to be acceptable and that Approval Voting is the clear winner when voting is done by Approval Voting. But, it is critical that we must not forget the significant concerns that have been raised about the type of winners that are selected when Approval Voting is employed.

J.-F. Laslier

I do not adhere to the Condorcet principle as a *normative* principle; if 49% of the population strongly prefer A to B and 51% slightly prefer B to A, I think that A is collectively preferable. My first best decision rule is thus utilitarianism, or “range voting”. But I found Approval Voting a very good practical mechanism to approximately achieve the utilitarian outcome. For the practice, I find that Condorcet-consistent procedures advisable, except in the extreme but important case of a society split in two. The best Condorcet procedure to me is the randomized procedure studied by B. Dutta, G. Laffond, M. LeBreton and myself under the name Essential set, but this rule was not proposed. In most cases, the Simpson rule (Minmax procedure) is a good way to select in the Essential set, like Kemeny, Coombs, and others. My preference was:

Range > Approval > various Condorcet methods among which I make little difference > Two round plurality > Alternative vote > Leximin > Majority Judgment > Plurality.

My guess was that, for this election, Approval would win, maybe challenged by Alternative vote (I was right !). Therefore I voted for Approval and Range. Here is my complete ranking, with my sincere utilitarian view scaled on the 0-100 scale:

Range (100) > Approval (99) > Kramer-Simpson (85) > Coombs (84) > Kemeny (83) > Copeland (82) > Nanson (81) > Black (80) > Borda (50) > Fishburn (21) > UncoveredSet (20) > 2-roundMajority (18) > AlternativeVote (17) > UntrappedSet (16) > TopCycle (15) > Leximin (10) > Majority Judgment (1) > Plurality (0)

M. Machover

I consider that decision about which voting procedure should be used must be governed by some meta-principle. I also consider that an appropriate meta-principle for the present hypothetical case is majority rule. I therefore gave my approval only to Condorcet-consistent procedures, selecting those that have additional desirable properties: Copeland's and Kemeny's procedures.

V. Merlin

While considering the question "what is the best voting rule that the city council of your town should use to elect a mayor?" my first reaction is that the procedure should be simple and easily understandable by the whole population of the city. The second question to answer is to which degree the Condorcet principle should be implemented. I do not adhere to the Condorcet principle, as a majority of 50% plus epsilon can impose a candidate which is the worst choice of the other voters, without considering compromise candidates. But at least, I do consider that a Condorcet loser should never be elected. Hence, Plurality rule is the worst system in the list.

So, I decided to advise Plurality with 2 rounds, Alternative Voting and Approval Voting. As long as there is a final duel, any elimination system using the plurality tallies will never elect the Condorcet loser. Plurality with two rounds and Alternative Voting are such systems. They are easy to explain, and have been implemented in different countries (France, Australia), with no major complaints. Moreover, Alternative Voting is hardly manipulable. I also consider that $k+1$ rounds before the final duel are better than k! Though I also voted for Approval Voting, it may be possible for it to select a Condorcet loser, if everybody just reports his first choice. But I think that the risk is quite limited, provided that a sufficiently large part of the population votes sincerely. Experiences show that voters tend also to approve more than one candidate. What would make me rank Approval Voting slightly below the two previous rules, is the fact that it has not been widely used in political elections. I felt that we still need more real life experiences to check that everything goes right with approval voting, but I am ready to give it its chance.

The simplicity argument goes against many Condorcet-consistent rules. Though Kemeny is an extremely elegant solution to the voting problem, it is rather sophisticated. For those who think that the Condorcet criterion should be implemented, I would recommend the Copeland method, which could be easily explained to the voters, as a tournament among the candidates.

At last, I fear that rules like the Borda count or Range voting could lead to undesired outcomes, when a fraction of the voters tries to manipulate it.

N. Miller

I cast approval votes for Approval Voting and Copeland. My votes did not reflect any general normative principle but rather my sense as to what would be both practical and reasonable for the type of election that Dan Felsenthal stipulated, namely the election of a mayor when a number of candidates are the ballot. A year ago I might have approved of AV/IRV also, but I now think that its problems are quite serious (even in practice, not just in theory). And, as a practical matter, my highest preference would be for Approval Voting, because it is simple to explain to voters, simple to cast votes, and simple to count. Moreover, most voters (in the US at least) would want to see some kind of vote totals in the newspaper the next day, which Copeland does not provide.

While Plurality lost our vote by a landslide, it works perfectly well in most US partisan general elections, since Duverger's Law works so powerfully that there are, literally or effectively, only two candidates in most such elections (the recent Senate contests in Florida and Alaska being notable exceptions). However, Approval Voting might be a definite improvement over Plurality in party primary elections and nonpartisan general elections (which is how many mayors are elected), where often three or more candidates are on the ballot.

Finally, voting procedures need to be evaluated not only in terms of their "static" social choice properties (e.g.,Condorcet consistency, monotonicity, etc.) but also in terms of their "dynamic" effects, e.g., incentives for candidate entry, candidate ideological positioning, etc., which affect the types of preferences profiles that are most likely to arise.

H. Nurmi

We were asked to propose voting systems that we could recommend or approve of to be adopted in the mayoral elections of our municipality. Recommend and approve of are two different — albeit related — things, but since we were asked to submit approval ballots, I felt encouraged to suggest more than one system (which I would NOT do if I were asked to recommend "a system"). I proposed Borda, Nanson and probably also Kemeny (someone may have preempted me on the latter, though). Anyway, my ranking is Nanson $>$ Kemeny $>$ Borda $>$ approval voting and these (as far as I now recall) were on my ballot. Nanson and Kemeny are both pretty resistant to misrepresentation of preferences and take into account a great deal of the preference information given by the voters. (One could also point out that they are Condorcet, but I'm not much moved by that property any longer: some systems are vulnerable to adding or removing or cloning alternatives (e.g. Borda) (as shown by Fishburn), others to adding or removing voters with completely tied preferences (Condorcet)(as shown by Saari). Overall, being based on strict majority principle is not a decisive feature in my book. Although it can be argued that it is preferable to be ruled by a majority than by a minority, I think one should also sail clear of the dictatorship of majority. They (Nanson and Kemeny) both do well in terms of several choice theoretic criteria. Borda's advantage is in intuitively plausible metric rationalizability: it looks for the closest (in terms of inversion metric) consensus profile (in terms of the first ranked alternative) and since we are looking for a single winner, this makes sense. Borda count also does well in minority protection (as shown by Nitzan). Approval voting was also on my ballot, not so much because of its choice-theoretic properties, but because of intuitive appeal of its results: it sounds nice to have a mayor who is deemed acceptable by more voters than any other. I must say, though, that the interpretation of "approvability" is not obvious (and this pertains to the interpretation of our balloting result as well). Does the fact that I approve of a candidate mean that I can tolerate him as the mayor without resorting to active resistance or does it mean that I positively support him/her? I think this is what makes the approval voting results hard to interpret, but I guess a mayor that is even tolerated by more voters than any other candidate has at least tolerable prospects.

F. Plassmann

I view voting as a useful mechanism for making collective decisions when unanimous agreement is not possible. Elections should generally be preceded by discussions about the candidates and the importance that the voters attach to the election. If a minority of voters feels strongly about some candidates while the other voters are almost indifferent between these candidates, then it should be possible for the minority to convince sufficiently many of the others to change their minds prior to the vote-casting process. (I believe that in cases of near-indifference, most people's desire to preserve social harmony trumps rent-seeking.) If it is not possible to change sufficiently many voters' minds, then I would interpret this as evidence that the intensity in preferences between the groups is not as disparate as it might appear. I therefore feel comfortable ignoring voting rules that take account of the intensities of voters' preferences.

I value the Condorcet principle, and I see the main issue as what we should do when there is no Condorcet winner. Apart from the fact that it is not Condorcet consistent, the Borda rule has many attractive properties. Thus my first choice is Black's rule, which seems to be least susceptible, among many popular voting rules, to a wide range of voting paradoxes and which has a very small frequency of ties (as preliminary research with Nic Tideman suggests). The discontinuity of Black's rule also makes strategizing difficult. However, the need to understand two separate evaluation criteria might make Black's rule too complicated for some voters. Voters will accept the outcome of an election only if they understand how the ballots are to be counted. Approval voting is very simple and avoids some of the most egregious shortcomings of the plurality rule. Thus I would endorse approval voting in situations when simplicity is important.

M. Salles

I voted for Approval Voting and for Borda. I share Jean-François' view regarding the difficulty concerning majority rule. However, I do not go as far as him and would not recommend ”range voting”. In case there are a sufficient number of candidates, the Borda rule proposes a way to deal somehow with intensity of preferences without going as far as ”Range Voting”. Also I think that the voting method must be simple enough to be understood by the quasi-totality of the voters, which might not be the case of the alternative vote system or Kemeny's rule.

N. Tideman

A group of experts on voting theory wanted to learn their collective judgments of a variety of voting rules. They decided (by something like acclimation) to proceed by using approval voting. I thought this was a reasonable way of learning the general level of support for different voting rules, as a prelude to future discussion. I would not have recommended approval voting as a way to make a collective judgment of which voting rule is best. That, I think, requires both more time and a procedure for ranking the options, so that direct paired comparisons can be made.

I am quite startled by the high level of support for approval voting as a way of electing a mayor. What I find particularly distressing about approval voting is that it requires a voter to decide whether to draw a line between generally acceptable and unacceptable candidates, or to leave that task to other voters and instead to draw a line between the very best and the close contenders who are not quite as good. I think that voters for a mayor should not be required to choose between drawing those two types of lines.

The relevant criteria for a voting rule for mayor, in my opinion, are:

- First, the capacity of the rule to gain the trust of voters. This depends on the reasonableness and understandability of the logic of the rule and the ease with which the counting process can be followed. Investigating this requires psychological methods as well as knowledge of the logic of voting procedures.
- Second, the likely statistical success of the rule in identifying the outcome with the greatest aggregate utility, under the assumption that voters vote sincerely. This is something that can be investigated by statistical methods.
- Third, the resistance of the rule to strategic voting. This too can be investigated by statistical methods.

It is my guess that the best rule, by some intuitive averaging of these criteria, is the Simpson rule. But the empirical work that would justify this guess remains to be done.

W. Zwicker

When I suggested we vote on voting rules and use Approval Voting, I thought the proposal could not pass – we'd surely split over the use of Approval Voting. At the time, however, our "rump session" discussion was stuck and it seemed that a conversational grenade might do more good than harm. I was very surprised that no one objected; some, as one might expect, were enthusiastic. Then I realized the exercise might be constructive if we could collectively endorse the principle that plurality rule was terrible... despite the stated goals of our workshop, I'd never thought it likely that we'd reach even a loose consensus on a single alternative. My own ballot approved a large number of rules, for two reasons: I doubt that the current state-of-the-art allows us confidently to select a small number of best rules, and my genuine indecisiveness was consistent with the best strategy for making plurality look bad. In terms of my specific approvals, it seems like false comfort to rely on any single absolute principle as a guide, when every choice of a voting rule entails trade-offs along many dimensions, about which our understanding is limited. For example, I feel the draw of Condorcet's principle but reject it as an absolute, in part because some recent results suggest trade-offs between that principle and any reasonable degree of decisiveness. I've come to view decisiveness as an under-valued trait – very important, though not decisively so of course. I did approve some Condorcet extensions, but not top-cycle, because of its striking indecisiveness. Mathematically, Kemeny is beautiful whereas Black is plug-ugly, but I swallowed hard, approved Black, and disapproved Kemeny (because Kemeny winner are rankings, not individual candidates, and I can imagine what would happen the first time some real world election yielded a tie among several rankings).

Appendix 2: 18 voting rules

In what follows, the "majority tournament" is the binary relation among candidates: "More than half of the voters prefer *a* to *b*". In that case we say that *a beats b* (according to pair-wise majority rule).

Approval voting [App]

Each voter approves as many candidates as she wishes. The candidate with the most approval is elected. See Brams and Fishburn (1983), Laslier and Sanver (2010).

Alternative vote [Alt]

Each voter submits a ranking (possibly incomplete) of the candidates. One first counts the number of times each candidate appears as top-ranked (his plurality score). The candidate with the lowest plurality score is eliminated. In a second count, the votes for this candidate are transferred to the second-ranked candidate (if any) on these ballots. The process is then repeated again and again until one candidate is ranked first by an absolute majority of the votes (original or transferred) is elected. See Farrell (2001), Farrell and McAllister (2006). Other names for this procedure or its variants: "Hare" system, "Single Transferable Vote", "Instant runoff".

Copeland [Cop]

Each voter submits a ranking of the candidates. For each candidate one computes his pairwise comparison score, that is the number of challengers this candidate beats under pair-wise majority rule. The candidates with the largest score are chosen. This Condorcet-consistent rule does not specify how ties (which are common when there is no Condorcet winner) are broken. See Laslier (1997). Other name: Tournament score.

Kemeny [Kem]

Each voter submits a ranking of the candidates. The rule defines a summary ranking as follows. For any ranking R of candidates one computes the sum, over all pairs (a,b) of candidates of the number of voters who agree with how R ranks a and b. Then R^* is chosen to maximize this total number of agreements. The elected candidate is the top-ranked candidate according to R^*. This procedure is Condorcet-consistent. See Young and Levenglick (1978), Young (1988). Other name: Median ranking.

Two-round majority [2R]

Each voter votes for one candidate. If a candidate obtains an absolute majority, he is elected. If not, a runoff election takes place among the two candidates who obtained the most votes. This rule is the most common rule throughout the world for direct elections, but it has seldom retained the attention of social choice theorists. See Lijphart (1994), Blais, Massicotte, and Dobrzynska (1997), Taagera (2007), Blais, Laslier, Sauger, and Van der Straeten (2010). Other name: Plurality with a run-off.

Coombs [Coo]

Similar to the Alternative Vote but, at each round, if no candidate is ranked first by an absolute majority of the ballots, the eliminated candidate is the one who is most often ranked last. This procedure is Condorcet-consistent in the single peaked domain. Coombs (1964)[7].

Majority Judgement [Bal]

Each voter grades each candidate according to some pre-specified finite grading scale expressed in verbal terms. For each candidate one computes his median grade. Among the candidates with the highest median grade, a linear approximation scheme (described in Balinski and Laraki, 2007) is used in order to choose the elected candidate. See Basset and Persky (1999), Gerlein and Lepelley (2003), Felsenthal and Machover (2008), Laslier (2011). Other names for this procedure or its variants: "Robust voting", "Best median".

Simpson [Sim]

Each voter submits a ranking of the candidates. The pair-wise vote matrix is computed. Then the chosen candidate is the one against which the smallest majority (in favor of another candidate) can be gathered. See Simpson (1969). Other names: "Minimax procedure", "Simpson-Kramer rule".

Borda [Bor]

Each voter submits a ranking of the candidates. For K candidates, each one receives $K-1$ points each time he is ranked first, $K-2$ points each time he is ranked second, etc. The elected candidate is the one who receives the largest number of points.

Black [Bla]

Choose the Condorcet winner if it exists and the Borda winner if not. Suggested by Black (1958).

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[7] Thanks to Dan Felsenthal for pointing to me details of this definition.

Nanson [Nan]

Each voter submits a ranking of the candidates. The Borda score is computed. Candidates with Borda score equal to or below the average are eliminated. Then a new Borda count is computed, on the reduced profile and the process is iterated. This procedure is Condorcet-consistent. See Nanson (1883).

Range Voting [RV]

Each voter gives to each candidate as many points as she wishes between zero and, say, 10 points. The elected candidate is the one who receives the largest number of points. Range Voting is not often considered in the voting rule literature since, from the theoretical point of view, it is essentially plain utilitarianism. See Arrow, Sen and Suzumura (2002), Dhillon and Mertens (1999), Baujard and Igersheim (2010) or the `rangevoting.org` web site. Other names for this procedure or its variants: "Utilitarianism", "Point voting" and in French: "vote par note", which just means "voting by grading."

Top Cycle [TC]

Each voter submits a ranking of the candidates. The majority tournament is computed. The Top-Cycle is the smallest set of candidates such that all candidates in this set beat all candidates outside this set. This Condorcet-consistent rule does not specify how ties (which occur when there is no Condorcet winner) are broken. See Schwartz (1972), Laslier (1997).

Uncovered set [UC]

Each voter submits a ranking of the candidates. The majority tournament is computed. A candidate a belongs to the Uncovered set if and only if, for any other candidate b, either a beats b or a beats some c who beats b. This Condorcet-consistent rule does not specify how ties (which occur when there is no Condorcet winner) are broken. See Miller (1980), McKelvey (1986), Laslier (1997). Other name (in the graph-theory literature): "Kings procedure".

Leximin [Lex]

Each voter grades each candidate according to some pre-specified grading scale. Each candidate k is evaluated according to the worst grade he received, say $g(k) = \min_v g(k,v)$. The elected candidate is the one with the best evaluation $g^* = \max_k g(k)$. If several candidates have the same evaluation g^*, the elected candidate is the one who receives g^* the least often. This rule is an important benchmark for normative economics. See Arrow, Sen and Suzumura (2002).

Fishburn

This choice correspondence is a variant of the Uncovered set which is useful when the majority relation contains ties (exactly as many voters prefer *a* to *b* than *b* to *a*). See Aleskerov and Kurbanov (1999).

Untrapped set

This choice correspondence defined by Duggan (2007) is a variant of the Top-Cycle which is useful when the majority relation contains ties (exactly as many voters prefer *a* to *b* than *b* to *a*).

Plurality

Each voter votes for one candidate. The candidate with the most votes is elected. This is the most common voting rule in the Anglo-saxon world and the literature is very large. Other name: First Past the Post.

Appendix 3: Statistical significance of the 3D representation

The method for spatial representation of data sets is derived from multivariate factor analysis. Given is a symmetric matrix of positive numbers, intended to measure the distances between the items, say $dist(c,c')$. If each item c is represented by a point $\phi(c)$ in the Euclidean space of dimension d one can compute the sum of the squares of the distances between the items:

$$\sum_{c,c'} dist^2(c,c'),$$

called the total variance, and compare this sum to the sum of squares of the distances between the corresponding points:

$$\sum_{c,c'} (\phi(c) - \phi(c'))^2,$$

called the explained variance). The best representation with d dimensions can be computed numerically using linear algebra. The quality of the representation is measured by the ratio between explained and total variance. This technique was used for Approval Voting data by Laslier and Van der Straeten (2004) and by Laslier (2006).

Of course the quality of the representation can only increase with the number of dimensions. In the text I show a 3D representation that explains about 90% of the the variance. In order to check whether this figure should be considered as large, I replicated the same computation on randomly generated data. Recall that, with the real data the explained percentages are, respectively 39, 66 and 90 for 1, 2 and 3 dimensions.

In a first test, suppose that each voter approves of each candidate independently with a probability p that corresponds to the average approval rate (here: $p = 78/(15*22) \sim .236$). Running 10.000 simulations I find that that the observed figures $(39\%, 66\%, 90\%)$ are respectively attained with probability .016, .005 and .0005. It is thus clear that our data set has much more structure than a totally random one, in which all candidates are alike, up to random fluctuations.

In a second test, suppose that we set the expected number of approval votes received by each candidate c to its actual value. So suppose that each voter independently approves of each candidate c with a probability $p(c)$ equal to the actual approving percentage of this candidate. For instance for the candidate *Approval Voting*, $p(\texttt{App}) = 15/22 \sim .6818$. We thus keep trace that some candidates are good and some are not, but we lose the correlation among candidates. In that case, I find that that the observed figures $(39\%, 66\%, 90\%)$ are respectively attained with probability .07, .07 and .03. Again one can conclude from this statistical test that it is not by chance that the real data set provides such large figures.

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